Statistical Inference for Subgroups Discovered by Machine Learning

Kosuke Imai

Harvard University

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Approaches to Subgroup Identification

1. Adaptive experimental design (Simon)
    - Goal: identify a subgroup with a positive average effect
    - Pre-specify strata and then drop those with little promise
2. Multi-period crossover trial (Ivanova)
    - Goal: identify the subgroup that maximizes the product of the average treatment effect and prevalence
    - Inference based on cross-validation and bootstrap
3. Estimation of the conditional average treatment effect (Lipkovich)
    - Goal: use machine learning to estimate the CATE
    - Identify a subgroup with large CATE estimates
4. Non-exchangeable subgroups (Schnell)
    - Goal: test consistency or heterogeneity among subgroups
    - Challenges of multiple comparisons in subgroup analysis

Subgroup Identification with Machine Learning (ML)

- Inference for subgroups discovered via a generic ML algorithm
  - cannot assume ML algorithms converge uniformly
  - avoid computationally intensive method
- Joint work with Michael Lingzhi Li
- Setup:
  - Conditional Average Treatment Effect (CATE):

$$\tau(\mathrm{x}) \;=\; \mathbb{E}(Y_i(1) - Y_i(0) \mid \mathrm{X}_i = \mathrm{x})$$

  - CATE estimation based on ML algorithm

$$s : \mathcal{X} \longrightarrow \mathcal{S} \subset \mathbb{R}$$

  - Sorted Group Average Treatment Effect (GATE; Chernozhukov et al. 2019)

$$\tau_k \;:=\; \mathbb{E}(Y_i(1) - Y_i(0) \mid c_{k-1}(s) \leq s(\mathrm{X}_i) < c_k(s))$$

for $k = 1, 2, \ldots, K$ where c_k represents the cutoff between the $(k-1)$th and kth groups

Statistical Inference for Subgroups

- An unbiased GATE estimator

$$\hat{\tau}_k \;=\; \frac{K}{n_1}\sum_{i=1}^{n} Y_i T_i \hat{f}_k(\mathrm{X}_i) - \frac{K}{n_0}\sum_{i=1}^{n} Y_i (1-T_i)\hat{f}_k(\mathrm{X}_i),$$

where $\hat{f}_k(\mathrm{X}_i) \;=\; 1\{s(\mathrm{X}_i) \geq \hat{c}_k(s)\} - 1\{s(\mathrm{X}_i) \geq \hat{c}_{k-1}(s)\}$

- Standard error based on Neyman's repeated sampling framework
  - random assignment of treatment
  - random sampling of units
  - random splits for cross-fitting

- No assumption about the properties of ML algorithms

Statistical Hypothesis Tests for Subgroups

1. Nonparametric test of treatment effect homogeneity:
   - Null hypothesis:

   $$H_0: \ \tau_1 = \tau_2 = \cdots = \tau_K.$$

   - Test statistic:

   $$\hat{\boldsymbol{\tau}}^\top \boldsymbol{\Sigma}^{-1} \hat{\boldsymbol{\tau}} \ \xrightarrow{d} \ \chi^2_K$$

   where $\hat{\boldsymbol{\tau}} \ = \ (\hat{\tau}_1 - \hat{\tau}, \cdots, \hat{\tau}_K - \hat{\tau})^\top$
2. Nonparametric test of rank-consistent treatment effect heterogeneity:
   - Null hypothesis:

   $$H_0^*: \tau_1 \leq \tau_2 \leq \cdots \leq \tau_K.$$

   - Test statistic:

   $$\left(\hat{\boldsymbol{\tau}} - \boldsymbol{\mu}^*(\hat{\boldsymbol{\tau}})\right)^\top \boldsymbol{\Sigma}^{-1} \left(\hat{\boldsymbol{\tau}} - \boldsymbol{\mu}^*(\hat{\boldsymbol{\tau}})\right) \xrightarrow{d} \bar{\chi}^2_K.$$

   where $\boldsymbol{\mu}^*(\boldsymbol{x}) = \operatorname{argmin}_{\boldsymbol{\mu}} \|\boldsymbol{\mu} - \boldsymbol{x}\|_2^2$ subject to $\mu_1 \leq \mu_2 \leq \cdots \leq \mu_K$.

Simulation Study

		$n_{\text{test}} = 100$		$n_{\text{test}} = 500$		$n_{\text{test}} = 2500$	
Estimator	truth	bias	coverage	bias	coverage	bias	coverage
Causal Forest							
$\hat{\tau}_1$	2.164	0.034	93.8%	0.041	95.0%	0.007	96.0%
$\hat{\tau}_2$	4.001	0.011	93.7	−0.060	94.4	−0.002	95.3
$\hat{\tau}_3$	4.583	−0.018	94.0	−0.003	96.4	0.020	95.8
$\hat{\tau}_4$	4.931	−0.077	94.6	0.001	94.3	0.003	95.6
$\hat{\tau}_5$	5.728	−0.058	96.0	−0.010	95.0	−0.009	95.2
BART							
$\hat{\tau}_1$	2.092	0.016	94.0%	−0.014	96.2%	0.009	95.8%
$\hat{\tau}_2$	3.913	0.127	95.1	0.028	94.0	−0.003	95.3
$\hat{\tau}_3$	4.478	−0.077	94.3	−0.041	95.0	−0.001	95.1
$\hat{\tau}_4$	5.042	−0.154	94.2	0.014	95.8	0.015	95.4
$\hat{\tau}_5$	5.881	−0.019	94.7	−0.019	94.4	−0.000	95.0
LASSO							
$\hat{\tau}_1$	3.243	0.028	94.1%	0.049	95.1%	0.003	95.1%
$\hat{\tau}_2$	3.817	−0.012	93.6	−0.013	94.5	−0.000	95.4
$\hat{\tau}_3$	4.318	−0.013	94.2	−0.002	94.5	0.010	95.0
$\hat{\tau}_4$	4.788	−0.041	94.0	−0.015	94.6	−0.001	94.6
$\hat{\tau}_5$	5.241	−0.046	94.4	0.021	95.1	0.002	95.3

Concluding Remarks

- Statistical inference for subgroups is challenging especially when they are discovered by complex machine learning algorithms
- The proposed methodology
  - no modeling assumption is required
  - any machine learning algorithms can be used
  - applicable to cross-fitting estimators
  - simulations: good small sample performance
- Ongoing extension: dynamic treatment settings
- Papers:
  - https://arxiv.org/pdf/2203.14511.pdf
  - Experimental Evaluation of Individualized Treatment Rules (JASA)
- Open-source software (R package):
  evalITR: Evaluating Individualized Treatment Rules